# OBSERVATIONAL PROSPECTS FOR EXTRA-GALACTIC MICROLENSING EVENTS

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## ABSTRACT

We consider the feasibility of directly observing gravitational microlensing in extra-galactic sources, whose stars are not generally resolved. This precludes use of the simple optical depth to microlensing formulation, which is applicable only to resolved stars. We, instead, extend this method to consider observational constraints, such as seeing, sky background and minimum detectable change in magnitude. Our analysis provides quantitative relations between these constraints and the expected observational results, event duration and number of detections per year. We find that an ambitious ground-based observer should detect several events per year in M31. We also consider detection of microlensing in visual binary galaxies. We find that, although these objects may present hundreds of events per year, extremely short durations would yield poor prospects for observation.

*Subject headings:* galaxies: individual (M31): other—gravitational lensing —dark matter

## 1. INTRODUCTION

Gravitational lensing and its effects are well-known (Refsdal 1964). However, meaningful astronomical results from this effect have been daunting until recently. A growing catalog of gravitationally lensed high red-shift objects assured us that the effect is observable, but until the last months of 1993, witnessing lensing by objects in our own galaxy had eluded us.

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In 1986, Paczyński proposed that galactic events would be detectable, but the probability of seeing a star during an event was only $10^{-6}$, which at that time seemed observationally prohibitive. Nonetheless, as digital detectors and faster computers became available, realistic projects emerged (Alcock, Axelrod, and Park 1989; Moniez 1990; Paczyński 1991). Now that these projects have detected several events at a reasonable rate, we at last have direct observational evidence of lensing in our galaxy. (Alcock *et. al.*, 1993; Aubourg *et. al.*, 1993; Udalski *et. al.*, 1994).

But the most exciting aspect of this work is the implication for the deflectors themselves. Though dynamical evidence of a dark matter halo in our galaxy was compelling (Binney & Tremaine 1987), no direct observations of this halo had been made. The microlensing events observed by the above-mentioned projects agree with a halo that consists at least in part of $0.01M_{\odot}$ - $0.1M_{\odot}$ objects, direct evidence at last of the dark matter halo, and important information on its constitution.

In this paper, we use such evidence for massive compact halo objects (MACHO's) to consider other possible observational targets for microlensing, particularly very nearby galaxies and nearby visual binary galaxies. Crotts (1992) proposed looking for lensing in M31, since we gain the advantage of having both our halo and its halo to produce events. Ballion, *et. al.* (1993) have used Monte-Carlo methods to estimate event probabilities in this regime. We address the problem analytically, with hopes to be quantitative with respect to observational constraints. Our analytical approach also allows us to extend our range beyond M31. Particularly, if other galaxies have halos similar to ours, as one would expect from dynamical evidence, observing a more distant galaxy through the halo of a nearer one should produce microlensing events.

## 2. THE MODEL

In estimating microlensing event probabilities, the seminal paper by Paczyński in 1986 covers the fundamental analysis thoroughly, and we shall not repeat its bulk here, but as it is widely accepted, we will assume its validity in our analysis. However, the situation concerning us is not quite the same one that faced him, so we offer a more specific analysis of the extra-galactic regime.

The most important difference between observing microlensing events in galaxies, and observing them in the Galaxy or LMC, is that we cannot necessarily resolve stars at the distances and stellar densities involved, which is done readily in the Galaxy and at LMC.

Thus, to see a single event, the star must not only be magnified significantly, but must be magnified sufficiently for significant magnification of the seeing element containing it. Figure 1 illustrates such a difference. The lower dashed line is the stellar flux, and the dotted curves show the time evolution of magnification of this star (described thoroughly in Paczyński 1991). Now suppose the star sits in a pixel whose magnitude is two less than the star's (6.3× as bright, shown in figure 1 as unity). For magnification of the pixel, we must consider the *sum* of the lensed star's brightness and the background's brightness, the evolution of which the solid curves show. For instance, in the top curve, where the star is magnified by nearly a factor of 10, the pixel is magnified by only a factor of two or so. Figure 2 quantifies these results for typical surface brightness and stellar magnitudes of very nearby galaxies, like M31. Even a very bright star in M31 at the Einstein ring of a Galactic MACHO would likely go undetected in a 17.5 magnitude background. For more typical stars, detection would be daunting even for high magnification events. The observational requirements would thus be far more demanding to see an event in such a regime.

We must, therefore, reevaluate our probabilities of detection according to these new observational constraints. We first consider the familiar single star case, then examine how the probability will be modified by placing the star in a bright background. Vietri and Ostriker (1983) give the ubiquitous equation for optical depth to microlensing which nominally describes the instantaneous probability of seeing significant magnification of one star against a black background.

$$P_* = \frac{1}{Area}\sum \pi R_{0,i}^2 = \int \frac{4\pi G\rho D}{c^2} dD \equiv \tau \tag{1}$$

This equation states that the probability is simply the ratio of the area inside the deflectors' Einstein radii, to the total area of the deflector plane. (If the reader is unfamiliar with the notation, he should consult Paczyński [1991], but here are the basic definitions:

$$R_0^2 \equiv \frac{4Gm_d D}{c^2}, \; D \equiv \frac{D_d(D_s - D_d)}{D_s} \tag{2}$$

with $D_s$ = source distance, $D_d$ = deflector distance and $R_0$ = the Einstein radius which is required proximity of a light ray to the deflector for significant gravitational bending.)

However, we have pointed out that an unresolved star requires more substantial magnification than a resolved star, so we must modify the areas in the sum accordingly. Since more magnification means smaller radii, and much less area, one may expect the probability for the required magnifications to be prohibitively small.

But consider Tonry's success at measuring small fluctuations among pixels in galaxies (Tonry and Schneider 1988). He measures tiny fluctuations to estimate distances to galaxies quite accurately. Could we then employ a similar method for detecting lensing events? Or would Tonry fluctuations confuse our search?

Figure 3 is a histogram plot of number of pixels vs. pixel surface magnitude from a Monte-Carlo distribution of stars in a $96 \times 96$ pixel region of sky at $0.5''/pixel$ and $\sigma_{seeing} = 0.5''$ (FWHM = $1.18''$ with sky background at $21^{mag}/(1'')^2$. The three simulated star-fields, with a luminosity function detailed in section 3, represent regions of surface magnitudes 20, 18.5, and 17.0 (factors of four in brightness from right to left) in a galaxy at $1Mpc$. Increasing the brightness means increasing the number of stars per pixel, which should mean less statistical fluctuation. And moving from right to left, one sees the expected sharpening of the distribution function. The graph has two implications for our purposes. The first is that even at low surface brightness, the distribution is quite narrow, implying that events of moderate magnification may be detectable above the pixel noise. The second is that, like the statistical fluctuations, events will be damped out as the pixel brightness increases, even as the number of stars available for lensing increases. We shall explore the manifestation of these competing effects in detail in section 3.

In any case, for a given apparatus, there is a minimum observable change in magnitude, or detection threshold, $\Delta m_0$. Since we require several points to claim an event, we define $\Delta m_0$ at the half-maximum magnification, $A_h$, which gives us this simple expression.

$$\Delta m_0 = -2.5 log \frac{A_h q f_* + f_{pix}}{q f_* + f_{pix}} \tag{3}$$

where the $f_*$ is the star's flux, $f_{pix}$ is the total flux of the pixel, including the sky background, and $q$ is the fraction of light in one pixel according to the point-spread function. With the required half magnification defined, we can find the maximum radius which produces it. Refsdal (1964) gives

$$A_h = \frac{u_h^2 + 2}{u_h \sqrt{u_h^2 + 4}} \tag{4}$$

where $u_h \equiv R_h/R_0$ is the dimensionless radius.

So, at a given radius, (3) and (4) allows us to find the minimum stellar flux [maximum magnitude, $m_V(u_h)$] required to produce an event of $\Delta m_0$. We now modify the optical depth equation, so that instead of calculating the areas inside the Einstein radius, we

calculate an effective area in which the magnification is sufficient to brighten the pixel by at least $\Delta m_0$.

Consider differential annuli in radius, each with its own maximum stellar magnitude, $m_V(u_h)$. We weight the differential area of the annulus, $2\pi u_h du_h$, with with the relative number of stars brighter than $m_V(u_h)$, according to the normalized luminosity function.

$$N(u_h) = \int_{m_V,min}^{m_V(u)} \phi(m'_V)dm'_V \tag{5}$$

We now integrate over radius to find the effective area.

$$S_{eff} = \int_{u_{h,0}}^{u_{h,1}} 2\pi u'_h N(u'_h)du'_h \tag{6}$$

Since the brightest stars require the least magnification, and hence allow the greatest radius, plugging the flux of the brightest stars into equations (3) and (4) determines the maximum radius, $u_{h,1}$. For the minimum radius, $u_{h,0}$, we use the approximate minimum radius criterion for point-source treatment given by Witt and Mao (1994) for a solar radius star in the source. This point-source criterion gives a radius significantly smaller than that of the dimmest stars at the required magnification. Since the criterion goes inversely with magnification, as does the radius (for moderate to large magnifications), we can be confident that the point-source treatment is valid.

Plugging into the optical depth formula we obtain the expected probability of a single star brightening a pixel by at least $\Delta m_0$.

$$\tau_{eff} = \frac{1}{Area}\sum R_{0,i}^2 S_{eff} = \int \frac{4G\rho D}{c^2} S_{eff}dD \tag{7}$$

By hypothesis, however, the number of stars per pixel is not usually one, so we multiply this probability by the number of stars per pixel to obtain the probability per pixel.

$$P_{pixel} = N_* \tau_{eff} \tag{8}$$

with

$$N_* = \frac{f_0[\exp_{10}(-0.4\mu_V) - \exp_{10}(-0.4\mu_{V,sky})]}{\langle f_* \rangle} \tag{9}$$

which is simply the flux of stars in the pixel, divided by the expectation value of the stellar flux.

Armed with this probability, we still lack a practical estimate for observational feasibility, because we have no idea how long an event will last. We thus commence analysis of event durations.

So as not to offend dynamicists, I will not assume any halo or disc model more complicated than estimating that deflectors velocities are random at $200km/s$ with respect to their parent galaxy. With no attempt to straighten out order unity effects, such as projections, or large effects such as inter-galactic motions, we shall proceed. (These results scale inversely with velocity, so pundits may make corrections as they deem necessary.)

As we know, stellar magnification depends only on radius (distance from the light ray to the deflector). A minimum magnification, $A$, thus corresponds to a maximum radius, $u$, within which all stars are magnified by at least $A$. For each circle of radius $u$, all stars whose impact parameter, $b$, is less than $u$ will pass through the circle twice along their paths. The path defines a chord on the circle, whose length is simple to calculate. Dividing by the velocity gives the crossing time from one side of the circle to the other.

$$t_{cross}(b) = \frac{2\sqrt{r^2 - b^2}}{v} \tag{10}$$

For our specific problem, $r = u_h$, and $v = R_0^{-1} \cdot 200km/s$ gives the result appropriate to our problem. Recall that $u_h$ is the radius where the pixel is at half-maximum magnification, so that $t_{cross}$ become the full-width at half-maximum duration, $t_{fwhm}$.

Since events have equal probability of occurring at all impact parameters, we perform the weighted integral over impact parameter to compute an expectation value for $t_{fwhm}$. First, we find the maximum magnification as a function of impact parameter using (4). ($A_{max}(b)$ and $b$ replace $A_h$ and $u_h$). The half-maximum magnification is $A_h(b) = [A_{max}(b) + 1]/2$, which we insert back into (4) to obtain the half-maximum radius, $u_h(b)$. To find the weighting factor, $N(b)$, we use (3) with $A_h(b)$, which gives the required stellar magnitude at each impact parameter. As before, the weighting factor is given by the integrated luminosity function. The limits of integration, $b_0$ and $b_1$, correspond to the half-maximum radius values as given for (6).

$$\langle t_{fwhm} \rangle(m) = \frac{2R_0(m) \int_{b_0}^{b_1} N(b)\sqrt{u_h^2 - b(u_h)^2}\, db}{\int_{b_0}^{b_1} N(b) db} \equiv T \cdot R_0(m) \tag{11}$$

Furthermore, since $R_0$ is explicitly dependent on mass, obtaining an expectation value requires an integration of $t_{fwhm}(m)$ against the mass-function, $\psi(m)$, which reduces to finding an expectation value of $\sqrt{m}$ (see [2]).

$$\langle t_{fwhm} \rangle = T \int_{m_0}^{m_1} \sqrt{\frac{4Gm'D}{c^2}} \psi(m') dm' \tag{12}$$

Having defined the probability of an event above the half-maximum detection threshold, and the expected time of such an event, the expected event frequency becomes $\langle P \rangle / \langle t_{fwhm} \rangle$. For useful units, we simply convert $\langle t_{fwhm} \rangle$ into years to find the expected number of events per year.

## 3. RESULTS

The equations in the previous section would be laborious to integrate analytically, and even if it were possible, such an analysis would limit flexibility in modifying the various parameters. We thus employed a fifth-order polynomial approximation to integrate them numerically according the method found in Press *et. al.* (1988).

Crucial to all results from our analysis is the large number of input parameters, which in principle could be manipulated to give practically any answer desired. We thus designed our analysis to allow flexibility with respect to these parameters, and if one thought our nominal values naïve, they could be easily changed to better values.

### 3a. M31

First, we consider prospects for M31. As pointed out by Crotts (1992), halo objects of either M31, or the Galaxy could provide lensing. For now, we study only the effect of Galactic deflectors; we will consider the effect of M31's halo on optical depth later. However, we shall presently explain why the event durations will be nearly identical for both cases. Witt and Mao (1994) give the explicit expression for duration, including source motion, deflector motion and observer motion. If one assumes that the velocities and distances of the Galactic deflectors relative to Earth are comparable to those of M31's deflectors relative to its stars, he can readily show that the two cases give the same duration to within a factor of $(1 \pm \delta v_{s,o}/v_d)$, with $|\delta| \lesssim 10kpc/770kpc$ and $v_{s,o}$, the relative velocity between the source and observer, of order $v_d$. Perhaps counter-intuitive at first, because of the slower angular

motion of M31's deflectors, the angular Einstein radius at M31 is also less by that same factor.

To estimate the parameters of the M31 case, we quite simply set the optical depth to $10^{-6}$, and the deflector mass to $0.02 M_{\odot}$. To emulate the stars of M31, we use the Hodge (1988) luminosity function, $\phi(M_V) \propto 10^{0.57 M_V}$ over a range of $-4 < M_V < 8$, which covers early B stars down to late K stars (Mihalas & Binney 1981). To estimate event duration, we set the deflectors in motion about both galaxies at $v = 200 km/s$.

Setting $\Delta m_0 = -0.1$ and FWHM seeing $= 1''$ gives figures 4(a) and 4(b), which show our results for event probability per CCD ($2048^2 \times P_{pixel}$; pixel width $= 0.44''$), and expected event duration in hours. The ordinate of figure 4(a) is the integrand in equation (6), just as the ordinate of 4(b) is the integrand in equation (10). Plotting this way allows one to see where the bulk of the contribution for probability and duration lies, and by definition, gives the final expected values as the areas under the curves. Both figures plot two families of curves labelled in surface magnitude. The solid family is for $\phi(M_V) \propto 10^{0.57 M_V}$, the dotted family for $\phi(M_V) \propto 10^{0.40 M_V}$.

Evaluating 4(a), we consider that at small radius (dimensionless radius at half-maximum magnification, $u_h$), the numerous dim stars will produce sufficient pixel magnification, but at large radius, only the few bright stars will produce sufficient magnification. However, the contribution to area is much greater for the high impact parameters, so it is hard to guess which effect will dominate. Figure 4(a) immediately resolves the confusion; the mild lensing of bright stars contributes the bulk of the probability, in agreement with Baillon *et. al.* (1993).

The shape of the dotted curves can be understood quite easily. Using 0.4 as the coefficient of $m_V$ in the magnitude function puts the luminosity function inversely proportional to flux, while for large magnifications the required flux goes linearly with impact parameter (see Section 2), and the two cancel. The extra factor of impact parameter from the logarithmic integration generates the linear slope of the curves, which turn over when the luminosity function runs out of stars on the bright end.

Examining the family of curves, we see that for low surface magnitudes (bright backgrounds) the event duration drops dramatically. Since the probability scales with the area under these curves (in linear space), the probability drops in the same way. Although putting more stars in the field by turning up the brightness increases the number of stars available for lensing, it more sharply stiffens the magnification requirements. In fact, observing a region at $\mu_V = 17.5^{mag}/(1'')$ increases event probability by more than a factor of ten compared to observing in a 14.5 magnitude region. Notice, however, a second effect

comes in at the very dim end. Here the $21^{st}$ magnitude sky has become comparable to the brightness in stars, so that fewer and fewer stars contribute to the total brightness. With less and less stars, but the same pixel brightness to overcome, the probability drops off quickly at the dim end.

In figure 4(b), the shapes of the curves are much the same as for 4(a), because as with probability, there is the luminosity function factor and two factors linear in $b$, the dimensionless impact parameter. Notice that again the curves roll over sooner for the brighter backgrounds, indicating longer expected durations for dimmer regions, until the sky background is reached. This faint regime contains the only significant difference between the figures. In 4(b), the sky changes the separation of the curves along the linear direction, so that the expected duration approaches an asymptotic value at the faint end, instead of dropping away altogether as does the probability. This is to be expected, because the number of stars lensed has nothing to do with the durations of the events.

Fixing $\mu_V$ at a constant value, we examine the effects of seeing and limiting detection threshold, $\Delta m_0$. In figures 5(a), (b) and (c), we have plotted respectively the event probability per CCD, the expected event duration, and event number per year, as functions of seeing, for several values of $\Delta m_0$. On each graph two values of $\mu_V$ are used, 19.0 (solid lines) and 17.5 (dotted lines). For surface magnitudes greater than about 19, the $21^{st}$ magnitude sky begins to interfere and decreases the probability substantially (recall figure 4[a]).

Figure 5(a) shows that event probability decreases by more than two orders of magnitude from perfect seeing to a seeing disc of a $2''$ diameter. Similarly, the curve families indicate that large detection thresholds hinder event detection; between $\Delta m_0 = -0.1$ and $\Delta m_0 = -0.5$, the probability drops by a factor of order 25.

As we move to the event durations, as seen in figure 5(b), we may wonder why duration depends on seeing at all. The durations of the stellar events are surely independent of observation, but the probability of seeing the longer, lower magnification events drops with seeing, so the expected event duration decreases with seeing, but not sharply. From perfect seeing to a seeing disc of $2''$, the expected duration decreases by about a factor of order 10, and as with the probability, event duration decreases with poorer detection thresholds, by a factor of order five. Ostensibly, good skies and sensitive detectors are of the essence for event detection.

Dividing probability by duration gives us the important quantity of number of events per year in figure 5(c). Together with 5(b), these plots probably provide the most useful reference for observers, since they give expected observational results as functions of

observational criteria. From these graphs, we see that for reasonable seeing, and detection threshold of 0.1 magnitudes, about ten three-hour events will occur each year (for a $2048^2$ CCD, $15' \times 15'$ field), which is probably too few to observe. However, Crotts (1992) points out that observing the far side of M31, through the bulk of its halo, increases optical depth by a factor of ten, which increases the number of events by the same factor. With good seeing, and ability to detect 0.1 magnitude changes, an observer may expect of order 100 events per year. Thus, if one observes on 120 nights, with six hours of data per night, he should see several events. Although somewhat more pessimistic than Baillon *et. al.* (1993), who estimate 50 detections per year, our results indicate a tenable number as well.

Having considered the effects of sky background, seeing, surface magnitude and detection threshold explicitly, we propose that microlensing would be detected in M31 under reasonable observational constraints. A particular method is that of Tomaney & Crotts (1994), who have demonstrated efficacy in detecting unresolved variable stars in M31. We submit that using a similar method would produce microlensing event detections (as Crotts [1992] points out). Our results should optimize such a search in both target region and exposure schedule. Namely, we find that the most productive regions of M31 will be the far side of disc in areas about a magnitude brighter than the sky background. However, large light gathering power, steady skies, and detectors capable of several exposures per hour, will be critical in detecting the bulk of events.

### 3b. Nearby Visual Binary Galaxies

With our analytical method, we have the added benefit of extending our range of objects beyond M31. Particularly, we can determine observational prospects for lensing events in visual binary galaxies. Figure 6(c) demonstrates that the number of events per year actually increases as the distance to the source increases, and thus shows the need to explore carefully the idea of observing microlensing beyond M31.

The main advantage with these sources is that instead of using the halos of our galaxy or the source galaxy, we use the halo of an intermediate galaxy to supply the deflectors. As equation 2 indicates, the geometry of lensing favors deflectors near the midpoint between the observer and the source. In figures 6(a), (b) and (c), instead of holding the optical depth constant, we have held the surface mass density in the deflector constant at $\Sigma = 200 M_\odot/pc^2$, and placed the deflector at half the source distance. We have labelled the five curves in surface magnitude, while setting the other parameters constant: $\Delta m_0 = -0.1$, FWHM seeing $= 1''$, and all other parameters are as in section 3a.

Although figure 6(c) presents an initially encouraging result, a quick glance a figure

6(b) shows that the durations could be prohibitively small at large distances. To make matters worse, $200km/s$ could be a drastic underestimate of the relative velocities in the deflector plane. Since the source, deflector and observer could all have motions at $\gtrsim 500km/s$, the durations could, in fact, be much less. Nonetheless, as I cannot estimate the light-gathering power of future telescopes, nor the speed of future detectors, I shall proceed to interpret our results briefly, while keeping with the very optimistic $200km/s$ relative velocity in the deflector plane.

Figure 6(a) shows the probability per CCD that an event is underway at a given time. At very small distances, we can resolve stars, and we recover the results for the resolved star case, where the probability increases as the third power of distance. Two of these factors are due to our constant solid angle pixels. The number surface density in stars must increase as distance squared for maintain constant surface magnitude, so over constant solid angle, the number of stars increases with distance squared. The third factor is the linear increase in optical depth from equation (2). Each curve, however, flattens when resolving stars becomes difficult due to the seeing disc. For the very bright regions, this happens quickly, but the peak for a $\mu_V = 20.5$ extends to around $1Mpc$. Beyond the peak, the probability falls off linearly, which can be understood as follows. The ratio of stellar flux to the constant pixel flux falls off as distance squared, so the required magnifications must increase with this factor for large magnification. Equation (4) shows that in this limit, radius goes as $A^{-1}$, so that $S_{eff}$ must go as $A^{-2}$. Having lost four powers of distance from our original three, we now understand the inverse relation of probability and distance. One further note on probability is that galaxies become smaller than the coverage of a single CCD chip, which will decrease the probability by distance squared once that limit is reached.

In figure 6(b), we see that the event duration converges to $\sqrt{D_s}$ for all $\mu_V$ as distance goes to zero. This result is expected from equations 2 and 11. The first departure from the convergent value is where the seeing discs first begin overlap, so that the required magnifications start rising. From this point to the turn-over, the curve depends on the luminosity function as bright stars are still effectively resolved, but the dim stars are slipping into oblivion. At the turn-over, even bright stars are no longer resolved, and the required magnifications grow rapidly. As with the probability, the required magnification increases with distance squared, and the required impact parameters (thus event durations) go inversely with magnification. Thus at large distances, duration contains two reciprocal factors of distance, but since the square root factor remains, the duration falls off with $D_s^{3/2}$.

Figure 6(c) is easily understood then, as simply the quotient of figure 6(a) and 6(b), so that in the end, the number of events scales with the square root of distance.

Considering figure 6(b) quantitatively, we see that out to about a megaparsec, the

duration slowly decreases from a few days to several hours, but beyond a megaparsec, duration decreases rapidly, down to minutes at $100Mpc$. If fast enough detectors and abundant photons existed, there would be hundreds of events detected per year, but with today's technology, we require that events be about an hour in length for multiple exposures per event.

Since duration scales with the square root of deflector mass, we may look to increase durations by increasing the deflector masses. We have used $0.02M_{\odot}$ deflectors, as before, so in order to increase the duration by a factor of ten, we would need to increase our minimum mass to $1M_{\odot}$, an unlikely prospect for halo objects.

We then ask if we could only use the brightest stars in the luminosity function, which have the longest possible duration. Returning to figure 4(b), however, we see that since the bulk of the expected duration already lies in the high duration end (lensing of the brightest stars), leaving little room for improvement in the luminosity function.

The only hope then, is to employ the very high mass end of the mass function. To get to $1M_{\odot}$, one suffers a factor $10^{-4}$ in probability, which is coupled with the factor of $10^{-1}$ due to the increased duration, to give a factor of $10^{-5}$ in event rate.

Space-based observation may offer some help, however. Figures 5(b) and 5(c) illustrate how the number of events increases by a factor of more than five for space-telescope seeing vs. ground-based seeing, while the duration increases by a factor of about two. These numbers still imply less than an event per year of one-hour duration. When we recall that our drastic underestimate of velocities pushes the durations up substantially, the picture is bleak indeed.

Nonetheless, for quantitative results, we did examine several actual cases. The tedious work of finding visual binaries within $100Mpc$, was greatly relieved by the kindness of Marc Postman who allowed us to peruse his unpublished data on such objects. Since distances are poorly known at this range, we assumed Hubble distances with $H_0 = 75km/s/Mpc$. Our search uncovered many candidate pairs, the best of which have $D_s \sim 50Mpc$, and $D_d \sim 10Mpc$. Such pairs give results similar to those mentioned above for both ground and space-based projects. Unless we were able to take many plates per minute, the prospects seem poor for observing microlensing events in this realm.

## 4. Conclusions

We have presented an analytical study to determine the observational prospects of observing microlensing in extra-galactic sources. We estimate that with reasonable ground-based equipment, several microlensing events will be found in M31 over the span of a year. Furthermore, we give detailed information on probabilities and durations of observed events according to a range of observational constraints. These data should help to focus an observing project onto the correct region of M31 according to equipment. In general, we find that the far side of M31's disc, in regions about a magnitude above the sky background, hold the most promise for event detection.

Nearby visual binary galaxies may also display individual microlensing events. However, even with space-based facilities, one would require many plates per minute, an unlikely prospect at the present.

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## FIGURE CAPTIONS

Fig. 1.— Magnification of a pixel as a function of time in a microlensing event. The heavy solid line is the total flux of the background. The dashed line is the relative flux of the star two magnitudes below the background. The dotted curves are the stellar magnification for impact parameters of $0.10R_0$, $0.25R_0$ and $0.40R_0$. The solid curves are the pixel magnification for the same impact parameters. The squares represent the half-maximum magnifications for each curve. Notice that even when the star is magnified nearly ten times, the pixel is only magnified by a factor of 2.

Fig. 2.— Magnification of pixel as a function of radius in a microlensing event. Each curve is labelled according to the stellar magnitude of the lensed star. The pixel background is at magnitude 17.5. The brightest star has $M_V = -3.2$ at M31. Events with bright stars at the Einstein radius, and dim stars $0.01R_0$, would likely go undetected.

Fig. 3.— Surface brightness fluctuations at $0.7Mpc$. With a stellar luminosity function of $\phi(M_V) \propto 10^{0.57M_V}$, $-4 < M_V < 4$ and $\mu_{V,sky} = 21$, we have plotted the distributions in pixel magnitudes for $\mu_V = 20$, 18.5 and 17. Since lower surface magnitudes require more stars per pixel, the width of the distribution decreases.

Fig. 4.— (a) Differential event probability per CCD with respect to logarithmic radius at half-maximum magnification, and (b) differential expected event duration with respect to impact parameter, for Galactic MACHO lensing of M31 stars (see text for details). The curves are labelled in surface magnitudes. The solid curves are for $\phi(M_V) \propto 10^{0.57M_V}$, the dotted curves for $\phi(M_V) \propto 10^{0.4M_V}$. The duration increases for lower surface brightness, as does total probability. The sky is at magnitude 21. These curves indicate that the bulk of duration and probability occurs in low magnification lensing of high luminosity stars.

Fig. 5.— (a) Event probability per CCD, (b) FWHM event duration, and (c) number of events per year, as functions of seeing, for Galactic MACHO lensing of M31 stars. The curves are labelled in detection threshold ($\Delta m_0$). The solid curves are for $\mu_V = 17.5$, the dotted curves for $\mu_V = 20.5$.

Fig. 6.— (a) Event probability per CCD, (b) FWHM event duration, and (c) number of events per year, as functions of source distance. The curves are labelled in surface magnitude in the source. The deflectors are in a galaxy half-way between the observer and the source ($D_d = D_s \div 2$).